Review Sheet II

Solutions

March 16, 2020

I. R is symmetric, so xRy implies yRx. R is antisymmetric so xRy and yRx implies x=y. Hence the only possible elements in R are xRx. R is the set of ordered pairs for which both hold. So it is one of the 8 subsets which contain some of the elements (x,x), is perhaps (a,a) and (c,c) etc..

II Take care of the lower inequalities by subtracting 2 from each of the 3 variables(donuts) and get $\binom{21}{19}$ We now have $y_1+y_2+y_3=19$ and $y_1 \leq 5$, $y_2 \leq 8$ and $y_3 \leq 23$ We solve first with $y_1 \geq 6$. Get$\binom{15}{13}$ Then $y_2 \geq 9$ Get $\binom{12}{10}$ Then $y_3 \geq 20$ Get $\binom{1}{-1}$=0. Next $y_1 \geq 6$ and $y_2 \geq 9$ Get $\binom{6}{4}$ The rest are 0. Ans=$\binom{21}{19}$-($\binom{15}{13}$ +$\binom{12}{10}$+0)+($\binom{6}{4}$+0+0)-0

III 1000-(31+10)+2. Inclusion exclusion. 31^2 is the last square less than 1000. 10^3 is the last cube less than or equal to 1000. Only 2^6 and 3^6 are both.

VI. Expand $(1+x)^n$ in the binomial expansion. Take a derivative. Substitute x=-1.

V. Expand $(x-1)^n$ in the binomial expansion and substitute x=3.

VI $\binom{n-1}{k-1}+\binom{n-2}{k-1}+\binom{n-3}{k-1}+\binom{n-3}{k}=\binom{n-1}{k-1}+\binom{n-2}{k-1}+\binom{n-2}{k}=\binom{n-1}{k-1}+\binom{n-1}{k}=\binom{n}{k}$ using Pascal's identity in every step.

VII. a. D_8 b. $\binom{8}{3}D_5$ c. 8! -D_8 d. 8!-D_8-$\binom{8}{1}D_7$ e. D_8/8!

VIII. $(1/(1-x^3))\ (x/(1-x^2))(\ (1-x^6)/(1-x))$

IX h_n stands for the number for a 1 by n board. $h_n=3h_{n-1}+3h_{n-2}$. $h_1=4$, $h_2=3$. Then $h_3=21$. Explanation: If the first is red there are 3 choices for the second and then h_{n-2} from there. If the first is blue, green or white, there are h_{n-1} choices for the rest.

X. Divide $1-x-x^2$ into 1 to get $1+x+2x^2+3x^3+5x^4+8x^5+$.... Fibonacci numbers. Actually f_n is the coefficient of x^{n-1}